ABSTRACT

Master's Thesis

FINITARY PERMUTATION GROUPS AND $\mathcal{S}$-GROUPS

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This thesis is a survey of A. O. Asar's and M. R. Dixon, M. J. Evans, V. N. Obraztsov, J. Wiegold's papers that are "On Finitary Permutation Groups" and "Groups that are Covered by Non-Abelian Simple Groups".

A group G is called FC-group if the conjugacy class of every element is finite. G is called minimal non-FC-group if G is not an FC-group, but every proper subgroup of G is an FC-group.

All minimal non-FC-groups that are different from their commutator subgroups are classified by Belyaev in 1978. Belyaev proved that if there exists a locally finite minimal non-FC-group G that is equal to its commutator subgroup is either $G/Z(G)$ is simple or G is a p-group for some prime p. In 1989, Kuzucuoğlu and Phillips showed that $G/Z(G)$ can not be simple. Therefore, if there exists a locally finite minimal non-FC-group G with $G = G'$ then it is a p-group for some prime p. But, existence of a perfect locally finite minimal non-FC-group has been a problem for 30 years.

F. Leinen and V. V. Belyaev are proved independently that if there exists such a group then it has a non-trivial representation into the group of finitary permutations on some infinite set Ω. So the existence problem of a perfect locally finite minimal non-FC-group turns out to investige to finitary permutation groups. Asar, in this paper, proved that there exist no such group if the following holds:

If G is a locally finite minimal non-FC-group and for every finite non-normal subgroup F of G there exists $y \in G \backslash N_G(F)$ such that $y^p \in FC_G(F)$, then G can not be perfect.

Let $\mathcal{S}$ denote the class of all groups that are the set theoretic union of their non-abelian simple subgroups. In chapter 4, some properties of such groups are examined. In this chapter, it is showed that if G is locally graded group, M is torsion-free radical of G and N is the normal subgroup of G generated by all the elements of finite order and if $N \neq 1$, then $M \leq N \leq G$, G/N torsion-free, N/M simple. In addition, if also $M \neq 1$, then every finite subgroup of G is cyclic or metacyclic.

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Key Words:

FC group, minimal non-FC-group, finitary permutation, almost primitive group, totally imprimitive group, locally finite group.