

ABSTRACT

M.Sc. Thesis

QUADRILATERAL NEC GROUPS

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\mathbb{H} , \mathbb{D} are two model of the hyperbolic plane they are called the upper-half plane model and the unit disc model. The isometries of the hyperbolic plane consist of homeomorphisms which are translations, rotations, reflections, glide reflections and limit rotations. Reflections and glide reflections are orientation preserving isometries while the others not. Orientation reversing isometries can be written as the composition of even number of reflections. Orientation preserving isometries can be written as the composition of the odd number of reflections. All isometries of the hyperbolic plane form a group under composition of functions and this group is denoted by $PGL(2, \mathbb{R})$. Any discrete subgroup of the group $PGL(2, \mathbb{R})$ with compact quotient space is an NEC group.

The reflections in the sides of a hyperbolic triangle with angles $\frac{\pi}{k}$, $\frac{\pi}{l}$ ve $\frac{\pi}{m}$ generate a NEC group where $k, l, m \in \mathbb{Z}^+$ and $k, l, m \geq 2$, and a NEC group obtained in this way is called a triangle NEC group. Similarly, the reflections in the sides of a hyperbolic quadrilateral with angles $\frac{\pi}{s}$, $\frac{\pi}{t}$, $\frac{\pi}{u}$ ve $\frac{\pi}{v}$ generate an NEC group where $s, t, u, v \in \mathbb{Z}^+$ and $s, t, u, v \geq 2$. Such a group is called a quadrilateral NEC group. Consider a hyperbolic quadrilateral with the above properties. If we consider the images of this quadrilateral under the elements of the quadrilateral NEC group generated by the reflections in the sides of the quadrilateral, we obtain a tessellation of the hyperbolic plane by quadrilaterals. The set of quadrilaterals with this property is called a tiling of the hyperbolic plane. Similarly, a tiling of the hyperbolic plane can be obtained by means of a triangle NEC group.

Let Γ and Γ' be quadrilateral and triangle NEC groups, respectively. If the group Γ is a subgroup of the group Γ' with index k , then any cell of the tiling of the hyperbolic plane by quadrilaterals can be divided into k triangles and two tilings of the hyperbolic plane with this property are called a divisible tiling pair.

In this thesis quadrilateral NEC groups have defined, some algebraic and geometrical relations between triangle and quadrilateral NEC groups have been investigated.

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Key Words

Hyperbolic space, Isometry, NEC(Non-Euclidean Crystallographic) group, Discrete subgroup, Triangle group, Quadrangle group, Tiling