ABSTRACT

INFINITE SYMMETRIC GROUPS

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This thesis is a survey of O. H. Kegel's paper "Regular Limits of Infinite Symmetric Groups". Also we read "Universal Groups" in Locally Finite Groups book, by written Otto H. Kegel and Bertram A. F. Wehrfritz and HNN-extension in "Combinatorial Group Theory" by written Roger C. Lyndon and Paul E. Schupp.

A group G is called locally finite group if every finitely generated subgroup is a finite. A locally finite group G is called universal if every finite group can be embedded into G and any two isomorphic finite subgroups of G are in G. Existence and basic properties of universal locally finite group are given by P. Hall's paper "Some Construction for Locally Finite Groups". P. Hall proved that there exist universal groups of arbitrary cardinal and also any two countable universal groups are isomorphic. And also he proved that universal group is simple and contains an isomorphic copy of countable locally finite groups.

In paper "Embedding Theorems for Groups" [5] proved that every countable group can be embedded in a group generated by two elements of infinite order. Also by this theorem, it is proved that there are $2^{\aleph_0}$ non isomorphic 2-generator groups.

Let G be a group and let A and B be subgroups of G with $\phi : A \to B$ an isomorphism. The group $H = \langle G, t \mid \phi(a) = t^{-1}at, a \in A\rangle$ is called an HNN-extension of G.

A sequence $\{\kappa_\nu\}$ of infinite cardinals with $\kappa_{\nu+1} = 2^{\kappa_\nu}$ for all ordinals ν and $\kappa_\lambda = \sup\{\kappa_\nu : \nu < \lambda\}$ for limit ordinal λ. Also a sequence $\{S_\nu\}$ of groups with $S_{\nu+1} := Sym(S_\nu)$ for every ordinal ν and $S_\lambda = \cup_{\nu<\lambda} S_\nu$ if λ is a limit ordinal. We have the set $\{(S_\nu, \rho_\nu) \mid \nu < \lambda\}$ is a direct system where $\rho_\nu : S_\nu \hookrightarrow Sym(S_\nu)$ is a right regular representation. We call the set $S_\lambda = \cup_{\nu<\lambda} S_\nu$ is a direct limit group of the direct system. S_λ is called regular limit group. [1] O. H. Kegel proved the basic properties of these regular limit groups. λ be a limit ordinal and a subgroup B of S_λ is called a bounded subgroup if $B \subseteq S_\nu$ for some $\nu < \lambda$. In chapter 4 of this thesis, some properties of bounded subgroups are examined.

G be a group and $H \supseteq G$ be a overgroup. The group G is existentially closed group in the over group H if every finite system Ξ of equations and inequations over G that is soluble in H has a solution in G. The group G is existentially closed if it is existentially closed in every overgroup. The regular limit group S_λ is a existentially closed group because it is a homogeneous and contains copy of every finitely generated group.

The group U is called universal if every group G with $|G| \leq |U|$ is isomorphic to a subgroup of U. Also for every infinite limit ordinal λ the regular limit group S_λ is universal.

Key Words
Infinite symmetric groups, existentially closed groups, universal groups, HNN-extensions